SECTION TWO
TEACHING/LEARNING STRATEGIES

This section of the Teachers' Guide focuses on some general ideas related to the teaching and learning of mathematics. This discussion covers some general strategies for teaching mathematics and catering to students' needs, procedures for using selected teaching methods, and strategies for developing the major emphases of the curriculum. The suggestions are illustrated by content specific examples that focus on the outcomes for Grades 5 and 6.

General Strategies for Teaching Mathematics

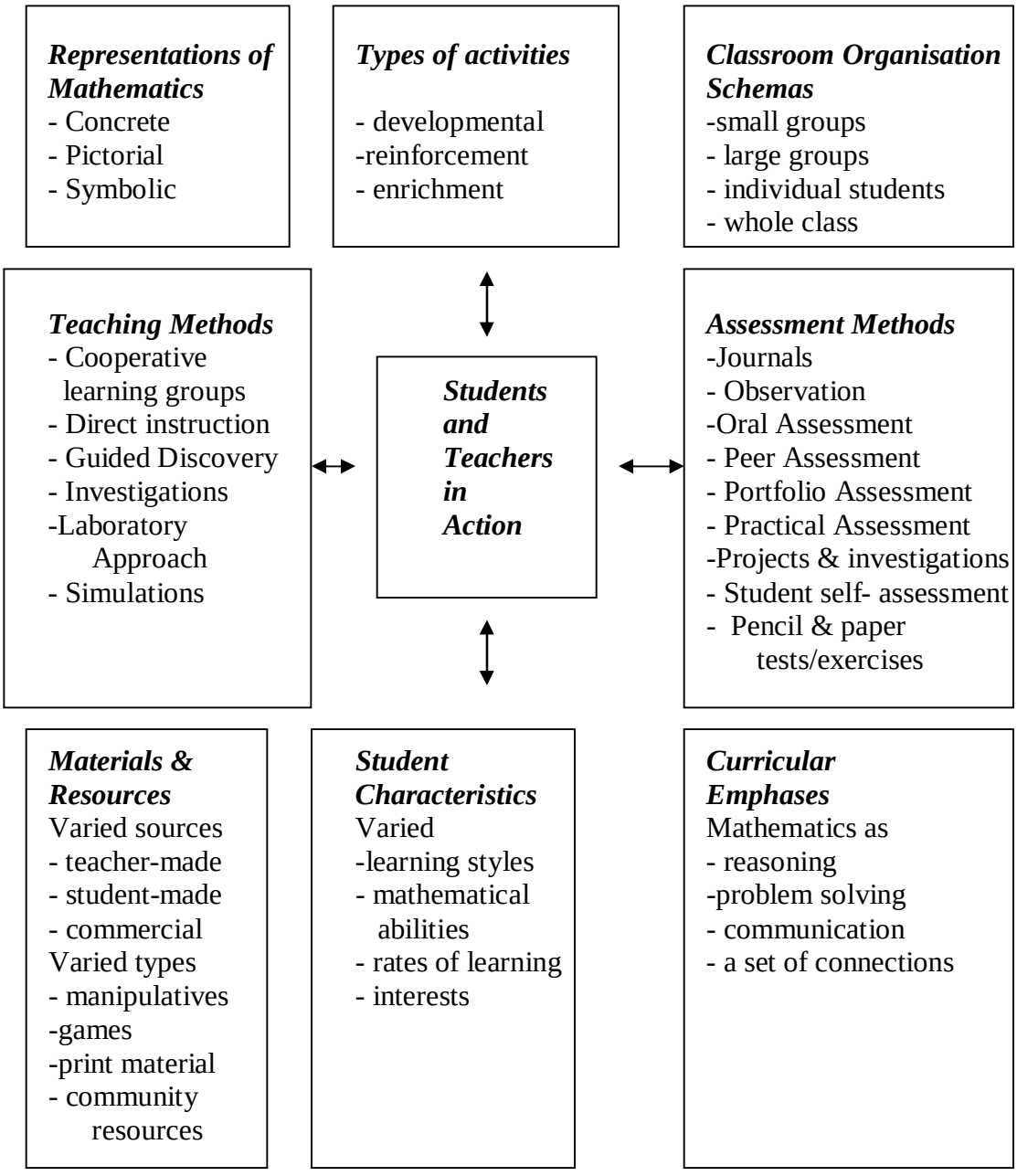
Although the students at the Grade 5 and 6 levels are more mature, physically and cognitively, they still need to engage with concrete and pictorial referents of the concepts, skills, and procedures that they are to learn. The activities must, however, be informed by a comprehensive conceptualisation of the experiences that are to be provided for the students in a lesson or unit of work. As you think about how you will organise your mathematics lessons, you could consider the following factors.

- **Teaching methods** – Which methods will be used during the lesson or unit?
- **Types of activity** – Which types of activities are appropriate for developing the selected learning outcomes?
- **Class organisation schemas** – How will the students be grouped for instruction?
- **Representation of mathematics** – Which modes of representation will form the basis of the instructional activities?
- **Curricular emphases** – Which curricular emphases will be the focus of the lesson or unit?
- **Students' characteristics** – How do my students differ? What are their specific needs? How do I cater to their needs?
- **Types of material/resources** – Which materials /resources are most appropriate for developing the selected outcomes? How will these materials/resources be used?

§ **Assessment methods** – Which assessment methods will be used during the lesson or unit? When and for what purposes will each of the methods be used?

Figure 7 outlines these factors and indicates some possible choices that you can make in relation to each factor.

Figure 7. Conceptualisation of mathematical experiences



It is important to note that the options for each factor may be combined in several different ways. Thus, for example, over the course of a lesson, you could use the following combination.

Conceptualisation of a lesson – An example

Teaching Method – Direct instruction and co-operative learning groups

Types of activities –Developmental activities

Class organisation schema – Whole class and small group instruction

Representation of mathematics – Concrete and pictorial representations of mathematics

Curricular emphases – A focus on problem solving and communication

Student characteristics - Attention to variation in mathematical abilities

Types of material – Commercial manipulatives and games developed by the students

Assessment methods – Observation, peer assessment, oral assessment (questioning)

Decisions about the various components of your lessons or units should focus on identifying those elements that are most appropriate for helping your students achieve the selected outcomes. As far as is possible and appropriate, you should also select a variety of elements for each factor to ensure that you cater to the mathematical and learning needs of your students. The implementation of your decisions should be guided by the following recommendations that are based on the learning theories proposed by several psychologists, e.g., Jean Piaget, Jerome Bruner, and Zoltan Dienes.

Guidelines for teaching mathematics

1. ***Make students aware that they can do mathematics and provide opportunities for them to be successful***.

   Students may require different paths to experience success in mathematics. It is therefore important that you identify how the students within your class differ and cater to these differences. Success could also be realised by making mathematics

meaningful and enjoyable for the students. The other recommendations provide some insights into how you might do so.

2. ***Ensure that students acquire a firm conceptual basis.***

   The learning outcomes have been sequenced across grade levels in such a way as to ensure this conceptual basis. You should avoid rushing exploratory and foundational work. Ensure that students spend as much time as necessary on exploring mathematics using concrete and pictorial representations before moving on to symbolic or abstract representations. This should ensure that they build up the necessary pre-requisites for future work.

3. ***Ensure that students perceive the need for new content.***

   This need can be developed through the use of activities or problems that cannot be completed without the new content. The involvement of students in these types of activities and therefore in the creation of mathematics is likely to increase their levels of motivation. It could also help students to remember and transfer what they have learned.

4. ***Develop new content by moving from the known to the unknown.***

   This suggestion is an extension of the previous one. Movement from known to the unknown can also be realised by:

   - initially using materials with which students are familiar before introducing novel materials.
   - drawing on the events and objects within students' experiences that embody the mathematical idea or procedure to be developed.
   - moving from the representations of mathematics given by these familiar objects and events to the abstract or symbolic representation.
   - using the students' current knowledge of the mathematical ideas to be developed as a basis for the teaching and learning activities.

5. Vary the nature of the materials.

Use a variety of materials to develop concepts, skills, and procedures. For example, in developing the concept of mean/average, you could use manipulatives and data that the students have collected. Additionally, in using each of the materials, you should ensure that the irrelevant mathematical features of the materials are varied. Thus in

using the concrete materials in the example above, the type of object does not determine the mean. This irrelevant feature should be varied, and therefore, a variety of manipulatives should be used.

6. Motivate students and sustain their interest to learn by making mathematics relevant to them.

You could make mathematics relevant to your students by, for example:

- Incorporating the use of real objects and realistic problems into the teaching of mathematics.
- Including games (e.g., Dominoes) that students are familiar with to explain and teach concepts.
- Using materials in the local environment to make instructional materials.
- Organising field trips or going on 'mathematics walks' to provide students with opportunities to discern how mathematics is used in a variety of real life situations and objects.

These general guidelines should form the basis of each lesson. However, there are some lesson-specific guidelines that you should consider. The following are some suggestions for developing your lessons.

Some lesson-specific suggestions

1. Share your lesson objectives with your students. Ensure that they know what they will be doing during the lesson. This may be done by:

    - § Posing a problem
    - § Engaging the students in an activity that stimulates their interest or creates a need for a fact, concept , skill, or procedure.
    - § Making a direct statement about the purpose of the lesson.

2. Link the content to be developed to relevant content that was covered in previous mathematics lessons or lessons in other subjects.

3. As students carry out their activities, ask them questions and encourage them to share their ideas or strategies. Let them examine each other's strategies to determine if the strategies are mathematically correct.

4. Use students' errors and misconceptions as positive teaching opportunities. Help them to recognise why their work or thinking is incorrect and to correct their errors and misconceptions.

5. Reinforce students' accomplishments.

6. Assess the students during the course of the lesson. Use the assessment information to modify the teaching/learning experiences as necessary.

7. Use your chalkboard carefully. Layout information on the chalkboard clearly, so that students can read it easily. Do not remove information that is written on the chalkboard too quickly. Give students sufficient time to read and respond to it.

8. Use the conclusion segments of your lessons to reinforce the knowledge, skills, or procedures that students developed during the lesson. You could, for example:
    - § Ask the students to present and explain their work or to explain what they learned during the lesson.
    - § Provide students with feedback on their performance.
    - § Link the current lesson to work to be covered in subsequent lessons.

9. Assign home work periodically. Homework tasks could require the students to:
    - § complete a practice exercise.
    - § gather information and materials for a subsequent lesson.
    - § work on puzzles.
    - § carry out an activity.

The above suggestions are intended to highlight the fact that children learn mathematics by engaging with it. Appropriate learning experiences provide students with opportunities to test and modify their existing mathematical knowledge through explorations that focus on mathematical ideas. Interaction between teacher and student and interaction among students are important features of these explorations. Students are encouraged to share their ideas, to explain the ideas they are developing and how and why they are developing these ideas. Their ideas then become the focus of discussions and further exploration aimed at assessing the extent to which these ideas are acceptable and correct.

Catering to Students' Needs

People are different in many respects. Thus it is possible to find a wide range of differences among the students in any one classroom. Students may differ in their rates of learning, levels of achievement, as well as their physical and emotional development. These differences may be related to developmental patterns, socialisation experiences, and personality attributes (Lemlech, 1998). Teachers need to be aware that these differences exist and that they may function to create special needs for individual students. Classroom teachers can address these differences and special needs to a certain extent in their classes. As a mathematics teacher, you can cater to some of these differences and needs through careful planning. However, you should also consult your Curriculum Officers for Mathematics and your Education Officer for Special Education for help, as necessary.

This section of the Guide provides some basic information to help you address the differences and needs within your classroom.

Catering to individual differences

One strategy for addressing differences among students is individualised instruction which involves the preparation of instructional programmes for individual students. Each programme takes into account the particular characteristics of the student for whom it was prepared. Another approach is to expose all students to a variety of

experiences, which may include some attention to individualised instruction. Given the context of today's classrooms, this second approach is viewed as more practical than the sole use of individualised instruction (Lemlech, 1998). It is also more effective (Schwartz & Riedesel, 1994).

Basically, you can provide a variety of experiences by varying several elements of the instructional environment. Here are some ways in which you can cater to the differences among your students.

Strategies for varying the instructional environment. You can vary the following aspects of the instructional context.

1. ***Tasks or activities***. The teaching/learning activities could require students to:
    - § Analyse a situation
    - § Use or illustrate a concept or skill
    - § Reflect on activities they have carried out
    - § Use manipulatives to explore mathematical ideas
    - § Read for information
    - § Discuss the ideas they are developing
2. ***Classroom organisation***. Use of a variety of grouping strategies allows students to interact with you and each other in different ways.
    - § Interact with students in different ways: individually, in small or large groups, or on a whole class basis.
    - § Use a variety of criteria to form groups, e.g., on the basis of interests, ability, levels of achievement, gender, etc. At some times they may be heterogeneous, consisting of students with different levels of one or more selected attributes. At other times, they may be homogeneous, consisting of students who are more or less at the same level of a selected attribute.
3. ***Content focus***. At times it may be necessary for all students to work on the same topic or task at the same level of difficulty. At other times, the content focus may be differentiated by varying the topic and/or the complexity of the tasks. The students may, for example, be set to work on the same topic at different levels of complexity or on a variety of topics.

4. ***Physical setting***. You can vary the physical setting in two ways.
    - § You can vary the location in which classes are held. Lessons may take place in the classroom or out of the classroom.
    - § You can also vary the environment within each location. Within the classroom, for example, some lessons may require students to work at their desks. For other lessons you may decide to use learning stations where students move from station to station and complete the task provided at each station. The lessons that take place out of the classroom could involve the use of the school compound or a field trip out into the community environs. For example you could use the school's playground to teach some of your lessons on measurement. You could also take students on a mathematics walk to note how geometry is used in their community.
5. ***Assessment strategies***. Students have preferred ways of expressing themselves. For example, some persons may express themselves better orally than in writing. Given that the aim of assessment is to find out what students know, the use of a variety of assessment methods will provide students with the opportunity to present their best work.

Students with special needs

For some students, their individual characteristics may be so distinctive that they fall into special categories, for example, gifted learners, slow learners, at–risk students. These types of students would need particular types of activities to ensure that they learn mathematics and develop to their fullest potential.

Some of your students might have been identified in the earlier grades as having special needs. Thus it is important to consult your students' previous teachers to obtain relevant information about their needs and the programmes that might have been developed for them. You can use this information to plan and develop suitable activities for those students. At the same time, you should be aware that you might have students in your class whose needs might not yet have been identified. If you think that there are students in your class who have special needs, you should seek help from the available

specialists. The following outline provides some information that will enable you to make a preliminary identification and to address the special needs of your students in you class.

Gifted learners. The mathematically gifted student exhibits special characteristics such as:

- A high level of mathematical achievement
- An ability to perceive mathematical patterns and generalisations
- An ability to reason analytically
- Flexibility in their thinking
- A higher level of persistence than is seen in other students
- Intellectual curiosity

(Schwartz & Riedesel, 1994)

Teachers need to nurture these characteristics of their mathematically gifted students through appropriately selected acceleration activities. Acceleration can be both vertical and horizontal. Vertical acceleration involves a movement through the regular curriculum at a faster rate than normal. The main advantage of vertical acceleration is that it allows gifted students to remain appropriately challenged and prevents loss of interest resulting from an otherwise lock-step curriculum.

Horizontal acceleration is sometimes referred to as enrichment. Enrichment is the expansion of the regular curriculum to include topics not generally encountered, and the study of many standard topics in greater depth. The following are some possible enrichment strategies.

Some suggestions for providing enrichment

1. Set up a mathematics corner or learning centre with a variety of materials. For example, you could include games, puzzles, manipulatives, and work and project cards. Students can work on the activities at their own pace during their free time or as part of a lesson.
2. Place a bulletin board in the classroom. Use it to display motivating information about mathematics and to present puzzles, problems, and other challenges to the students. The students can also have an input by presenting their challenges to their classmates. You can also display the students' responses and solutions on the bulletin board.
3. Use teaching methods that provide students with opportunities for learning independently. Two examples of these methods are the laboratory approach and investigations.

The slow learner. Children come to school with various types of intelligences. Those students who are deficient in mathematical-logical intelligence may have difficulty learning mathematics. Note that the level of difficulty could vary from topic to topic. Some characteristics of the slow leaner are:

- They have short attention spans.
- They may forget information easily.
- They have difficulty in interpreting written material.
- They tend to enjoy 'drill-type' activities.
- They lack the motivation to participate in class activities.

(Schwartz & Riedesel, 1994)

Slow learners can learn mathematical concepts and skills. However, they need more time than other students. They also need structured learning experiences with constant repetition. Some suggestions for teaching the slow learner follow.

Some suggestions for teaching the slow learner

1. Focus on one new idea at a time and provide instruction in small increments.
2. Provide simple tasks that the students can easily repeat until they develop an understanding of the idea.
3. Ensure that instructions and explanations are explicit. You may need to repeat and model ideas several times.
4. Select materials and models that are simple and can be easily used by the students.
5. Provide appropriate supplementary materials for the students. You can present the slow learner with appropriate reinforcement and challenge by including a wide range of materials and activities in your mathematics corners and on the bulletin boards.

At-risk learners. Learners who are at risk are not necessarily mentally challenged but have problems which are associated with the learning environment. This category includes students with perceptual deficits, memory deficits, integrative disorders, attention disorder. Van de Walle (1990) provides a synopsis of these problems.

Perceptual deficits may be visual or auditory. These students may have difficulties in:

- Sorting out component parts of what they see or hear. For example, they confuse the steps in simple algorithms or confuse directions that consist of several parts.
- Discerning differences in the things that are seen and heard. These students may hear ‘fifty’ instead of ‘fifteen’. They may also write their numerals backwards.
- Organising their work on paper. They may confuse orientations of up, down, left, and right. They also have difficulties in understanding pictures of three-dimensional shapes.

Memory deficits are related to the ability to recall information. This deficit can also be visual or auditory. Some children may have more difficulty recalling things they have seen than what they have heard and vice versa. Some specific types of memory deficits are:

- Short-term memory deficit. These students have difficulty recalling information a short time after it was seen or heard.
- Long-term memory deficit. These students have difficulty retaining information over days or weeks.
- Sequential memory deficit. These students have difficulty retaining an order to a sequence of events or a series of steps in a procedure.

Integrative disorders involve an inability to make connections or associations. These disorders are sometimes referred to as relation-thinking disorders. These disorders may be exhibited as follows.

- Closure difficulties - Students who experience this type of difficulty do not see or hear things in the same obvious groupings or categories as those in which they were presented. These students will fail to see simple related patterns or to make connections between two similar problems or situations.
- Receptive language deficiency – Students who have this deficiency may not understand the language associated with mathematics. They may also have problems with visual presentations.
- Expressive language deficiency – This deficiency encompasses difficulties associated with verbal expression. Students with this deficit may have difficulty putting what they know into words.
- Abstract reasoning deficiency- This deficiency is related to the inability to construct relationships and connect abstract ideas with symbols. Students with this deficiency would have a greater reliance on models than other children.

Attention disorders are those deficits related to processing information. Students with this disorder are easily distracted and unable to focus for any length of time on one task. They give too much attention to unimportant details. This disorder may be characterised by:

- Impulsivity – a tendency to respond to situations without much thought. For example, some students may habitually respond to questions immediately without thinking about what the question requires of them.
- Compulsiveness – a tendency to continue on a trend of thought in spite of evidence that the line of reasoning is inappropriate.

In teaching students who exhibit one or more of these deficits, it is important to teach to the students' strengths. You should use teaching strategies that allow the students to use the abilities that they possess. Given that you may have students with a variety of disorders in your class, it is also important that you vary the strategies to meet the strengths of all students. At the same time, do not hesitate to seek professional help from your Education Officers for Special Education.

Teaching Methods

Six teaching methods will be discussed in this section. These methods are:

- Co-operative learning
- Direct instruction
- Guided discovery
- Investigations
- Laboratory approach
- Simulations

In each case, information on the purposes and procedures associated with the method will be presented.

Co-operative Learning Groups

Co-operative learning is a small-group teaching strategy in which students work in small groups to discuss and complete tasks. This method is particularly useful in developing students' level of achievement and affective attributes (Orlich et al., 1990). It is also an appropriate strategy for slow learners. Specifically, co-operative learning groups may be used to help students:

- Review home-work
- Develop concepts and skills
- Explore the application of mathematical concepts and skills

- Complete laboratory tasks

You could use the following questions to help you decide if co-operative learning is appropriate.

- Is the content such that students can learn it from each other?
- Would student interaction be helpful?

The basic structure of a lesson that utilises co-operative learning groups is as follows.

Using co-operative learning groups

1. Before the students are organised into groups, there should be a class session to prepare them for the activities. In this session, you can:
   - Present or help students generate problems or questions to be investigated.
   - Help the students to clarify the purposes and nature of the group tasks.
   - Present and discuss the materials to be used. The students can also select appropriate materials.
2. Once the students have been placed into groups you should monitor their work and progress. Ask questions and give advice as appropriate. Your role is to facilitate the group activity.
3. After the group session, there should be a class session in which the students present and discuss the results of their group effort. This session should also be used to ensure that the students have developed the intended outcomes that were the focus of the group activity and to provide feedback to the students.

The effectiveness of the co-operative learning group strategy is dependent on the students' ability to work in groups. Therefore it is essential that you teach the students group process skills related to discussion, listening, and group leadership (Arends, Winitzky, & Tannenbaum, 1998; Orlich et al., 1990).

To promote discussion skills, you can:

- § Draw the students' attention to discussion rules such as taking turns to speak, listening to the person who is speaking, using a tone of voice to allow all persons in the group to hear what is being said.
- § Model the ways in which they can acknowledge another's ideas.

To promote listening skills, you can:

- § Discuss appropriate non-verbal listening behaviour with the students.
- § Use short, simple directions. Write detailed directions on the chalk board.
- § Play games such as 'Pass the message'. In this game, one student gives another student a message. The message is then passed around the class from student to student. The initial and final versions of the message are then compared.

To promote leadership skills, you can :

- § Talk with the students about the types of activities that group leaders will have to carry out. Appropriate activities at the Grade 5 - 6 level are: keeping the group on task, ensuring that all members of the group have a chance to contribute, keeping a record of the group members' contributions, co-ordinating the preparation of the group report or product, and ensuring that the group's product is of an acceptable standard.
- § Monitor the students during the group sessions, and provide feedback on the group leaders' strengths and weaknesses.

Decisions about the size of the groups should be made in relation to the tasks to be performed and the students' level of development of group process skills. The students' first encounter with co-operative learning groups could involve two students working together. As they acquire proficiency in group process skills, the group size could be increased to 4 to 5 persons, dependent on the task to be performed.

The groups' composition could be based on a variety of criteria, for example, levels of achievement, gender, interests, and social backgrounds. The groups should be heterogeneous. However, you should strive to maintain a balance in the criteria that are used to form the groups. For example, if level of achievement is used as a criterion for

group composition, there should be about the same number of students from the above average, average, and below average achievement levels.

Evaluation of co-operative learning groups should involve an individual and group focus. Each student should be evaluated on the basis of his/her input into the group and personal achievement. The group's process and the product of the group activity should also be identified and assessed. Feedback should help individuals and the group to improve their performance. Some appropriate methods for assessment are observation, oral assessment, and pencil and paper exercises.

Direct Instruction

Direct instruction is a form of expository teaching. The lesson is controlled by the teacher who presents information, asks questions and elicits responses from the students (Lemlech, 1998). This teaching strategy is useful for developing skills, for example, using measuring instruments, and understanding of simple information. It is less effective for outcomes that involve problem solving.

This strategy works to the benefit of slow learners who favour a more structured learning environment, but it may frustrate gifted learners.

Direct instruction consists of five basic steps (Orlich et al., 1990).

The five basic steps of direct instruction

1. *Introduction*. Here, the teacher informs the students of the focus of the lesson and the expectations for student performance.
2. *Demonstration*. In this phase, the teacher provides an example and highlights the purpose of the content to be developed. The teacher may also explain how this content is linked to what the students already know.
3. *Presentation*. The teacher develops the concepts, skills, or procedures that are the focus of the lesson. Instruction is sequential, moving step by step from simple to complex, known to unknown. Concrete or pictorial materials are used to help students develop the relevant content. The students' understanding is checked through carefully phrased and sequenced questions.
4. *Guided Practice*. The students complete a short practice exercise under the careful supervision of the teacher. Oral questioning, small group work, or written exercises on the chalkboard may be used.
5. *Independent Practice*. The students complete a practice exercise individually. The teacher monitors the students and provides feedback in the form of positive reinforcement or a short review related to any areas of difficulty.

To ensure that you use direct instruction effectively, you should take note of the following. At all phases of your lesson:

- Ensure that all students are attentive.
- State your instructions and directions clearly. Repeat any new or unusual aspects of the activity.
- Monitor students' understanding constantly.
- Follow-up on those students who are experiencing difficulty in understanding the work. Identify and clarify their areas of difficulty.
- Motivate your students by giving them positive reinforcement.

Guided Discovery

Discovery learning involves students in the use of mental strategies to gain knowledge or develop concepts that were previously unknown to them. The guided discovery method requires that students are guided through a series experiences that are designed to help them discover the concepts, skills, or procedures that are to be learned.

The teacher's role in a guided discovery lesson is to:

- Provide the necessary materials
- Provide the students with sufficient examples of the mathematical idea to be developed so as to enable the students to form a generalisation. Note that it is difficult to generalise from 2 or 3 examples, and therefore several examples should be used.
- Ask questions related to the content to be developed.
- Refocus the activity, as necessary.
- Encourage the students to make observations and conjectures, form generalisations, and state their findings.

(Lemlech, 1998)

More specifically, you may use the following steps to plan and conduct guided discovery lessons.

Organising for guided discovery

1. Identify the content (concepts, procedures, or generalisations) that you want the students to develop.
2. Design a series of activities or tasks to lead the students to develop the desired content. The activities should involve the use of various types of materials. Use questions to guide the students through the task.
3. Ask the students to write or state what they have learned.
4. Have the students test their discoveries. They could, for example, consider the following question. 'Is there an example that does not fit with the generalisation we have formed?'
5. Provide exercises to allow the students to apply and test their generalisations.

The following is an example of an activity that could be used in a discovery lesson.

An example of a guided discovery-oriented activity

Here are some numbers.

5, 7, 11,12, 13, 26, 31, 36, 37, 42, 59, 61, 89, 91, 97

1. Write down the factors of each of the numbers.
2. Sort the factors of each number into two groups, factors that are composite and factors that are prime.
3. What is common about all of the factors that are prime numbers? These factors are called prime factors.
4. Write down a statement that explains what prime factors are.

Investigations

An investigation is a form of discovery. It is a more open-ended process than guided discovery (Schwartz & Riedesel, 1994). In a guided discovery lesson, the teacher selects the learning outcomes and structures the activities so that students can develop the desired content. However, there is more student input into an investigation. Additionally the problems or tasks may be more open-ended. The following are the characteristics of an investigation.

Characteristics of an investigation

The students:

1. Pose or select the initial problem. A common source of problems is the difficulties that students may be experiencing.
2. Select the materials and procedures that they will use to solve the problem.
3. Collect any necessary data, make and test their conjectures, and form their generalisations.
4. Decide on the manner in which they will communicate the results of the investigation.

While the students are carrying out their investigation, they should discuss their procedures and record their results. Records could take a variety of forms, for example, charts, bulletin board displays, oral presentations, and booklets. During the investigation, you should monitor their progress and provide help as necessary.

Some examples of investigations which may be carried out at the Grade 5 - 6 level are as follows.

Some examples of investigations

1. Williams' Ice-Cream Parlour sells 8 different flavours of ice-cream. When customers buy cones they can select two flavours. How many different combinations of flavours can they choose from?
2. Are the flags of the countries of the Organisation of Eastern Caribbean States (O. E. C. S) symmetrical?

Laboratory Approach

The laboratory approach is a form of discovery teaching. In using this approach, the teacher presents a problem which requires the students to use previous knowledge and experiences to develop a new idea. Characteristics of the laboratory approach are as follows.

- The students use experimentation and exploration to solve the problem.
- The students exchange and combine ideas to develop the new content.
- Within a class, a variety of procedures may be used to solve the problem.
- A variety of results may be generated.

The following are some examples of activities for the laboratory approach.

Some laboratory activities

1. What is the area of your hand?
2. The length from your elbow to your wrist is more or less the same as the length from your heel to your big toe. Three times the measurement of your wrist to your elbow is almost the same as your waist measurement. Three times the perimeter of your head is more or less equal to your height.

   Is this true for you?

Simulations

A simulation is the presentation of a problem, event, or situation that mimics reality. In a simulation, the students are active participants who engage in an activity to learn new content or apply previously acquired knowledge and skills (Orlich et al., 1990). Simulations may be used to:

- stimulate interest
- provide information to students, and
- enhance skill development

The following are some guidelines for using simulations.

Developing and using simulations

1. Select the learning outcomes that you want students to develop.
2. Examine real life situations to determine the situations in which the content or skills associated with the outcome are likely to be used. Use this real life situation to develop a simulation that is appropriate for your students' level of development and the outcome that the students are to achieve. This phase of development should also consider the purposes of the simulation. If the intention is for students to apply and extend concepts and skills then the procedures should involve a practice element and a challenge.
3. Inform students of the procedures for engaging in the simulation.
4. Let the students engage in the simulation. Monitor their performance through observation and questioning.
5. Follow-up the simulation session with a discussion of the mathematical ideas that the students used during the simulation.

A useful example of a simulation is the class shop. To set up this simulation, you can ask the students to bring in empty containers or discarded household items. Let the students price the items. For the most part, the prices should be realistic. At the same time, you should also consider the use of a variety of computation strategies. If the prices are too large to allow mental computation or pencil and paper computation procedures, then a calculator may be used. The students will have to decide and discuss which transactions require the use of a calculator and give reasons for their decision.

To play shop, the students take turns at being the shopkeeper. The other students would then go to the shop to buy goods. The shopkeeper must give change, if necessary. This simulation provides an opportunity for developing decision-making skills. If each student is given a certain amount of money then they would have to decide which items they can buy with the available money. The shopkeeper would need to decide which combinations of coins and notes to use to ensure that he/she does not use up the available money in giving out change.

This simulation can be extended to include measurement, for example, measurement of mass and length. In purchasing the goods, the students could request specific measures (e.g., 1 kg of rice or 1.5 m of ribbon) which the shopkeeper would have to measure. It may also be extended by combining it with another simulation, such as running a household on a specified amount of income. Here the students will need to decide how to budget their money to meet expenses such as paying bills and purchasing food and other necessary items from the class shop.

The methods described in this section are intended to provide you with a range of strategies you can use to help students learn mathematics. The selection of the method to be used in a lesson should be guided by a consideration of the outcome to be developed, the characteristics of your students, and the purposes of the various teaching methods.

Developing the Curricular Emphases

This section considers some strategies for developing the curricular emphases of communication, logical reasoning, problem solving, and building connections in mathematics

Developing Communication Skills

Language is a means of communication that consists of four important elements: reading, listening, speaking and writing. Within the mathematics classroom, language is sometimes used in a limited manner in that communication tends to be dominated by the teacher. It also revolves mainly around oral exchanges/discussions and written exchanges involving the use of a series of symbols, e.g., number sentences (Sheffield & Cruishank, 2000). However, all language components can, and should, be used at all levels of the primary system.

The effectiveness of these communication-oriented activities can be enhanced by attention to the characteristics of mathematical language. Mathematics is a language with its own vocabulary, symbols, and syntax. Examples of this vocabulary include the names for concepts and procedures. Symbols are used as short forms of the vocabulary. A symbol could have different interpretations in different contexts. Moreover, there are rules that guide how the vocabulary and symbols may be used.

Therefore, to facilitate students' use of this language in their explanations and reports, you should help them develop an understanding of its vocabulary and symbols. This attention to language is also important because students encounter mathematical language in their everyday experiences at home and in their communities. Analysis of the various uses of the terms would provide students with a means of developing and checking their understanding of the terms.

Here are some activities that you can use to incorporate communication into your mathematics classes.

Communication in the Mathematics Classroom

Students can:

1. Write out their solutions to exercises and problems.
2. Help in developing charts that summarise the main ideas, skills, or procedures developed during mathematics lessons.
3. Read, listen to, and talk about stories or current events with a mathematical theme.
4. Analyse newspaper and magazine articles and advertisements with a mathematical theme.
5. Explain how and why they did various activities, e.g., classifying, measurement of their desks.
6. Comment on their classmates' solutions to problems. They can identify the similarities and differences between two students' responses to a task.
7. Describe an activity they engaged in.
8. Keep journals in which they express their thoughts about mathematics and/or explain how they solved selected problems.
9. Put on dramatic presentations or draw diagrams that illustrate concepts that they have learned.
10. Write poems that provide an explanation of a concept.
11. Interview family members to determine how they use mathematics.
12. Explain what they understand by the mathematical terms they have encountered. Some useful questions to consider are:
    - § How is the use of this term in mathematics (e.g., table in Statistics) related to its everyday usage?
    - § How is it different from every day usage?
    - § How is it related to other mathematical terms?
    - § What does the term mean?
13. Examine and talk about the mathematics presented in their workbooks/textbooks.
14. Make up riddles related to mathematical concepts.

Developing reasoning skills

Students' reasoning abilities could be developed through engaging them in a variety of thinking processes. These processes form the basis for many mathematical activities, and they help students to develop a sound knowledge of concepts and an understanding of the procedures that they carry out.

Examples of these processes are: Observing, inferring, comparing, classifying, sequencing, generating and testing hypotheses, and validating and justifying conclusions (DES, 1987; Hatfield, Edwards, and Bitter, 1999; Sheffield & Cruishank, 2000).

Observing. Activities that involve observation require students to use their senses to identify and describe the characteristics of objects and the steps in procedures. These descriptions can be presented orally and/or in writing. The following are some activities related to observation.

Observation activities

Students can:

1. Examine models of mathematical concepts and describe or sketch them.

   For example, in teaching geometry, students could examine the nets of three-dimensional shapes and describe them. Students could also be required to look at geometrical shapes and make sketches of them.

2. Observe demonstrations of mathematical procedures then describe the procedures.

   In measurement, students could observe demonstrations of the measurement of capacity of several containers then describe the procedure for doing so.

Inferring. Activities designed to develop inferential skills require students to form generalisations about objects or events on the basis of their observations. In carrying out these types of activities, students use the knowledge they would have acquired from previous observations to complete the task at hand. Following are some examples of inferential tasks.

Making inferences

Students can:

1. Identify examples of concepts given an explanation of the concept.

   For example, the following activity can be used to reinforce the attributes of three-dimensional and plane shapes.

   Place students in pairs. Let one student in the pair describe a shape. The second student then identifies the shape that was described. Ask the students to exchange roles and repeat the activity.

   Note that this activity could also be used in reinforcing number concepts.

2. Use the characteristics of an object to identify other examples of that object.

   The following examples relate to the attributes of three-dimensional shapes.

   § The students can observe and analyse examples of the nets of cuboids, and then sketch other shapes that are nets of the cuboid.

   § Place several three-dimensional shapes in a bag. Have students feel the bag and name the shapes that are in the bag. Give the students a set of objects and have them select those shapes that are in the bag from the set.

Comparing. Comparison of objects and procedures goes a step beyond observation. Activities that require students to compare objects, in essence, ask them to use the results of their observations to identify how the objects are alike and different.

Activities focussing on comparison

Students can:

1. Identify how examples of a concept are alike.
   In studying the different types of numbers, the students could be provided with examples of a particular type of number, e.g., prime numbers. They could then be asked to identify the common characteristics of the set of numbers.
2. Identify similarities and differences between related concepts.
   You could ask students to identify the similarities and differences between various
   - shapes, for example, cubes and cuboids.
   - graphs, for example, bar graphs and pictographs.
   - types of numbers, for example, odd and even numbers.
3. Identify the similarities and differences between two different ways of solving a problem.

Classification. Classification is the sorting of objects into groups according to some systematic basis. Initially, classification activities could include the criterion for sorting the objects. However, students should also be encouraged to develop and explain their own criteria for sorting objects.

Classification activities

Students can:

1. Group objects according to a given criterion.
   For example students could be asked to group a set of containers of different shape and sizes according to their capacity.
2. Sort objects in as many ways as possible.
   For example, students can be given a set of numbers to group them in as many ways as possible, for example, as odd, composite, and/or square numbers..

Classification activities can be used to introduce students to new ideas. For example, by examining the groups that students formed in Activity 1 above, you would be able to check their level of understanding of capacity. Additional activities could then be developed to help those students who have misconceptions.

Sequencing. Sequencing involves ordering a set of objects according to given criteria. Students may for example, write a set of numbers in descending order. The analysis and development of class inclusion relationships are also related to sequencing. These types of relationships identify whether one set/type of objects is subsumed in another. An example of this type of relationship involves squares and rectangles: 'All squares are rectangles'.

The following are some examples of sequencing activities.

Sequencing activities

Students can:

1. State the next or missing item in a sequence. E.g.

   Fill in the blanks in the sequence below.

   1, 4, 9, ___, ___, 36, 49, ___, ____, 100

2. Continue a pattern. E.g.,

   State the next number sentence in the sequence below.

   11 x 11=121

   11 x 111 = 1 221

   11 x 1 111 = 12 221

3. Investigate class inclusion relationships.

   At the Grade 5 - 6 level, students can examine the similarities and differences between concepts, and investigate whether an object can belong to more than one group. For example, they can explore whether a three-dimensional shape can be classified as a cuboid and a cube; or whether a number can be odd and composite.

Generating and testing hypotheses. Students generate hypotheses when they formulate statements of the form 'if … then …'. For example,

'If I add two odd numbers, then my answer will be an even number.'

'If a three-dimensional shape has 6 faces, then it is a cuboid.'

Tests of these types of statements involve a search for examples that do not fit with the statement. Thus to test the second hypothesis, students could select examples of three –

dimensional shapes with 6 faces and examine them to determine if they are all cuboids. In so doing, some students might include cubes in their set of examples. Such a selection could lead students to consider whether cubes are cuboids, and whether the hypothesis, as stated, is acceptable or needs to be modified to read:

'If a three-dimensional shape has 6 rectangular faces, then it is a cuboid.'

Evidently, investigations related to this thinking process can stimulate in-depth discussion of mathematical ideas.

Activities: Generating and testing hypotheses

Students can:

1. Formulate statements of the form 'if ….then….' and discuss what these statements mean.
2. Identify those statements that represent plausible relationships.
3. Use a variety of strategies to determine if the conclusions in their statements follow from the premises.
4. Make speculations about situations in their everyday life.
   For example, 'The most popular recording artiste among Grade 5 students is ….'
   Let students talk about how they will find out if their guess/conjecture is correct.

Validating and justifying conclusions. Validation of conclusions involves ensuring and proving that results are correct (Sheffield & Criushank, 2000). Thus students validate conclusions when they check that their answer to a computation exercise (e.g., 15 +75 or 511+ 24) is correct. Students justify conclusions when they give reasons for their conclusions. Note that their reasons will be age related and dependent on their level of mathematical competence. For example, by Grade 6 the students should be able to use their insights into the relationship between addition and subtraction to justify

the answer to an addition statement. To help students develop their abilities to validate and justify their conclusions, you will need to focus on the language of mathematics and the connections that exist among the concepts and procedures.

Types of activities for validating and justifying conclusions

Encourage students to:

1. Work backwards, retrace the steps they took in completing a task.
2. Check the reasonableness of their answers.
3. Give reasons for their answers.
4. Check that their reasons are acceptable.

The activities related to reasoning skills are useful in that they enhance students' understanding of mathematics. They provide opportunities for reinforcing knowledge and skills that students have acquired and developing new ones. Additionally, they help students to make connections among several concepts, skills, and procedures.

Developing Problem Solving Abilities

First, consider some definitions. A problem may be defined as a perplexing situation or question for which there is no immediate method of solution (Hatfield, Edwards, & Bitter, 1999). Problem solving is the process of identifying and removing the obstacle in a situation or obtaining an answer to a question. A problem is classified as routine if it involves the use of mathematical procedures in much the same way as they were taught, and non-routine if the strategies or mathematical procedures to solve it are not immediately evident (Reys, Suydam, & Lindquist, 1984).

Examples of routine problems are many of the text or workbook problems as described above. For example students may be given the following problem.

The average of 5 numbers is 25. When a sixth number was added to the list, the average changed to 30. What number was added??

Non-routine problems tend to be open-ended in that they may be solved in several ways or have several answers. The following is an example of a non-routine problem involving addition of two-digit numbers.

Use the digits 1,2, 3, 4, 5, 6, 7, 8, and 9 to form two two-digit numbers with a product that lies between 250 and 300. Write down all the pairs of two-digit numbers that you can get.

These definitions are important to the discussion of problem solving, because the definition of a problem and the existence and need to solve non-routine problems imply a consideration of several aspects of the problem solving process.

There are several ways in which you can incorporate problem solving into your mathematics classes: teaching about problem solving, teaching for problem solving and teaching via problem solving (Schroeder & Lester,1989). Descriptions of these approaches follow.

Approaches to problem solving instruction

Teaching about problem solving: This focus involves teaching students a problem solving process such as that identified by Polya (1973). The students are also taught a variety of strategies that would enable them to develop solutions to problems. This approach involves discussion and analysis of how problems are solved.

Teaching for problem solving: The focus is on how mathematical ideas can be used to solve routine and non-routine problems. The aim of this focus is to ensure that students can transfer what they have learned from one problem context to others. The students are given several types of problems involving the concepts and skills that they have learned. This is accompanied by analysis of the solutions to these problems.

Teaching via problem solving. Problem solving is used as a means of developing mathematical content. Students develop the mathematical concepts, skills, or procedures in the process of solving problems.

Each of the above approaches should be used in your mathematics classes. At times it may be necessary to help students develop an understanding of the problem solving process and problem solving strategies in order to enhance their likelihood of success at solving problems. At other times, it may be necessary to focus on applying particular mathematical ideas to solve problems. Thus, you would need to organise a variety of problem solving experiences for your students.

The following are some guidelines for enabling students to become proficient problem solvers.

The problem solving process

Help students to use a problem solving process. The basic stages of this process as outlined by Polya (1973) are as follows:

1. *Developing an understanding of the problem.* This may involve reading and paraphrasing the problem, summarising the information, identifying relevant and irrelevant information, listing the known and unknown information, or comparing the given problem with other similar problems they have solved.
2. *Developing a plan to solve the problem*. Here students explore and identify strategies that may be used to solve the problem. Possible strategies are:

    - Classifying information
    - Searching for a pattern
    - Drawing a picture, diagram or table
    - Guessing and checking, and improving the guess
    - Assuming a solution and working backwards
    - Using a formula or writing an equation
    - Solving a simpler problem
    - Acting out the problem
    - Accounting for all possible solutions

3. *Implementing the plan*. At this stage, students use their selected strategy to solve the problem. They work carefully, check their work, present their solutions clearly, and persist until they obtain a solution. It is important to help students recognise that, as in real life, attempts at solving a problem may not always be successful. Thus it may be necessary to devise another plan then continue the process.
4. *Evaluating the solution.* This stage involves checking the reasonableness of the answer and determining whether another solution exists. Students may also be guided to compare the current solution to the solutions for other similar problems. In so doing, they may gain insights into appropriate strategies for solving various types of problems.

Problem posing. Problem posing or creating problems is an essential aspect of mathematics. It can motivate students and help them develop a positive attitude towards problem solving. It can also enhance their problem solving abilities. Some suggestions for developing students' problem posing abilities follow.

Creating/Making up problems

Encourage pupils to:

1. Create or design problems for their class to solve. To stimulate this type of activity, you may, for example, ask the students to:
   - § Use their own experiences to restate or rewrite given problems in their own words (Jurdak & Shahin, 1999).
   - § Make a problem in their workbooks simpler or more difficult.
   - § Use information they gathered in statistical or measurement activities or investigations to generate problems.

   These types of activities can help students develop problems that have some meaning for them.

2. Analyse the problems that they develop.

   This analysis can focus on whether the problem:
   - § can be solved.
   - § is clearly stated.
   - § has too much information.
   - § Is easy or difficult/challenging.
   - § has several possible solutions.

   This type of involvement helps to lessen students' fears and develop their interest in problem solving.

In integrating problem solving into the mathematics classroom, students should be encouraged to compare and discuss their solutions to problems. As students talk about their solutions they are likely to explore and use a variety of related facts, concepts and skills. Moreover, the discussions can help students expand their repertoire of problem solving strategies. As they listen to each other's solutions, they can develop an awareness of the contexts in which certain strategies are useful and those in which they are not. This awareness will help them to apply the strategies successfully. Thus, these discussions are useful in that they provide opportunities for learning.

Making connections in mathematics

Three types of connections that students can make are: linkages among concepts and procedures, between mathematics and other subjects, and linkages between mathematics and real life.

Linking Mathematical Ideas

Help students to:

1. Relate new concepts to those they already know.

   For example, students will most likely recognise and use the relationship between addition and multiplication if their initial activities related to multiplication guide them to consider how repeated addition situations could be represented more efficiently.

2. Develop an understanding of the relations that exist among mathematical ideas, e.g.:

   - The four basic operations
   - Units of measurement
   - Number representations, e.g., whole numbers, fractions, and decimals
   - Three-dimensional and plane shapes

3. Use the results of activities in one topic as the basis of the activities in another topic.

   For example, the data collected during activities focussing on Statistics could become the basis for computational activities.

There are several opportunities within the learning outcomes for students to relate mathematics to several other subjects. Here are a few examples.

Connecting mathematics and other subjects

Students can:

1. Compare the meaning of terms that are used in mathematics and other subjects.
2. Compare and contrast the procedures that are used in Mathematics and other subjects.

   Measurement provides opportunities for connecting mathematics with other subjects. Students may, for example, explore how measurement principles are used in the measurement of weather elements such as rainfall.

3. Apply procedures learned in mathematics to other subjects and vice versa.

   For example, in developing outcomes related to the collection of data in Statistics, you may refer students to procedures they used in Social Studies.

Connections between mathematics and real life may be developed in a variety of ways.

Connecting mathematics and real life

You can:

1. Use real life situations as bases for activities. For example activities related to measurement of mass and capacity could involve the measurement of the ingredients for a recipe. Television and radio advertisements for sales and other promotions could be used to develop reasoning skills.
2. Ask students to explain how they use mathematical ideas outside of the classroom.
3. Send notes home to parents indicating the concepts you are teaching and suggesting ways in which the parents can reinforce the ideas students are learning.

   Some examples of possible reinforcement activities are as follows.

   - To reinforce computations involving money, parents can involve their children in the preparation of their weekly or monthly budget.
   - Parents can allow their children to explore the relative costs of products that they buy to determine which is the cheapest or most expensive. For example, the children can determine which brand and which size of laundry detergent is the best buy.

Activities focussing on connections in mathematics are essential because they help students to develop and consolidate their understanding of mathematical ideas. Additionally they also help students to develop an appreciation for the importance and relevance of mathematics in their lives. Such benefits could enhance students' attitude

towards mathematics. It is therefore important that teachers use every opportunity available to emphasise these relationships.

The suggestions and activities presented in this section of the guide are but a few of the possible experiences that can be provided for students. Yet they reflect the wide range of ideas that teachers should attend to in developing teaching/learning experiences. Your task is to focus on these ideas and select the strategies and activities that are most appropriate for your students.