SECTION SIX
Teaching/ Learning Geometry

General Overview

The learning outcomes for Geometry focus on the development of an understanding of the properties of three-dimensional and plane shapes and how these shapes are related to each other. The study of three dimensional shapes focuses on the following shapes: the cube, cuboid, cylinder, cone and sphere. The regular two-dimensional shapes to be studied at this level are: the square, rectangle, triangle, and circle. Irregular shapes as well as geometric figures such as the line, line segment, point, and angle are also included.

The study of geometry at Grades 5 and 6 is aimed at helping students to:

- Refine their understanding of the attributes of three-dimensional shapes
- Compare and contrast three-dimensional shapes
- Draw three-dimensional shapes
- Draw and make nets of three-dimensional shapes
- Construct three-dimensional shapes
- Represent and identify two dimensional shapes
- Classify two-dimensional shapes
- Compare and contrast the properties of two-dimensional shapes
- Develop an understanding of several basic geometric figures
- Explore the concept of symmetry
- Develop an understanding of simple co-ordinate systems

In teaching geometry, it is useful to consider the ways in which students learn geometry. One useful explanation of students' development of geometric ideas is the van Hiele theory (van Hiele, 1984). The theory indicates that, students pass through a sequence of five levels of thinking. Each level is based on a different way of thinking about geometrical ideas, and they are more related to instructional experience than biological maturation (Wirszup, 1976). A student reasons at a particular level because of the type or quality of instruction received. Table 2 presents a synopsis of the levels.

Table 2. Description of the van Hiele levels of thinking

Level	Description	Example of student behaviour
1	Students judge and operate on shapes and figures according to their appearance	Students can identify a shape, e.g., a rectangle, because each looks like an example they were shown.
2	Students analyse shapes in terms of their components. They compile the properties of a class of figures.	Students can identify properties of shapes, e.g., rectangles. They can provide the names of shapes given their properties.
3	Students formulate and use definitions. They follow deductive argument	Students can define concepts in several ways. They can identify class inclusion relationships.
4	Students are able to state formal proofs.	Students can prove theorems. E.g., they can prove that the sum of the angles in a triangle is 180 degrees.
5	Students can analyse various deductive systems.	Students can use formal logic symbols in proving theorems

Van Hiele (1984) indicated that movement through the phases is facilitated by specific types of activities. These activities are characterized by:

- Teacher-student interaction, where through discussion the teacher assesses how students interpret the vocabulary associated with the topic to be studied.
- The use of carefully sequenced exploratory activities that help students to focus on the specific concepts, skills, or procedures to be developed.

- Activities that emphasise observation and enable students to refine their use of vocabulary.
- The completion of multi-step tasks that are discovery-oriented and which enable students to develop their reasoning abilities.
- Analysis of the relations that exist among the concepts, skills, and procedures that students have developed. This analysis is to enable internalisation of the associated geometric ideas.

Evidently, then, the emphases included in the OECS Primary Mathematics Curriculum are consistent with the instructional guidelines provided by the van Hiele theory. You should note that the learning outcomes in Geometry focus on the first three levels of thinking.

Geometry, therefore, provides a good opportunity for you to engage students in exploratory and investigative work. The following are some suggestions for activities.

Guidelines for teaching Geometry

1. Activities should require students to observe, describe, compare, classify, draw, and construct shapes.

2. Initial activities should focus on identifying the features of individual shapes, e.g., shape, size, and number of sides, number of angles or vertices. Over time you should guide students to describe the attributes and defining features of the various shapes.

3. Once students have been introduced to two or more shapes, you should encourage them to compare and contrast them to identify similarities and differences. These types of activities should be continued throughout the grades.

4. Activities that involve drawing should also be an integral part of the study of geometry. For example, students could:

    - § Draw three-dimensional shapes from several orientations. They could place a three-dimensional shape on their desk and draw the shape as it appears when they look down on it or when they look at it from a side.

    - § Sketch examples of plane shapes by looking at them and by following given directions; e.g., 'Sketch a two-dimensional shape that has three sides and two of these sides are equal in length'.

5. Activities involving construction should require the students to make models of various shapes.

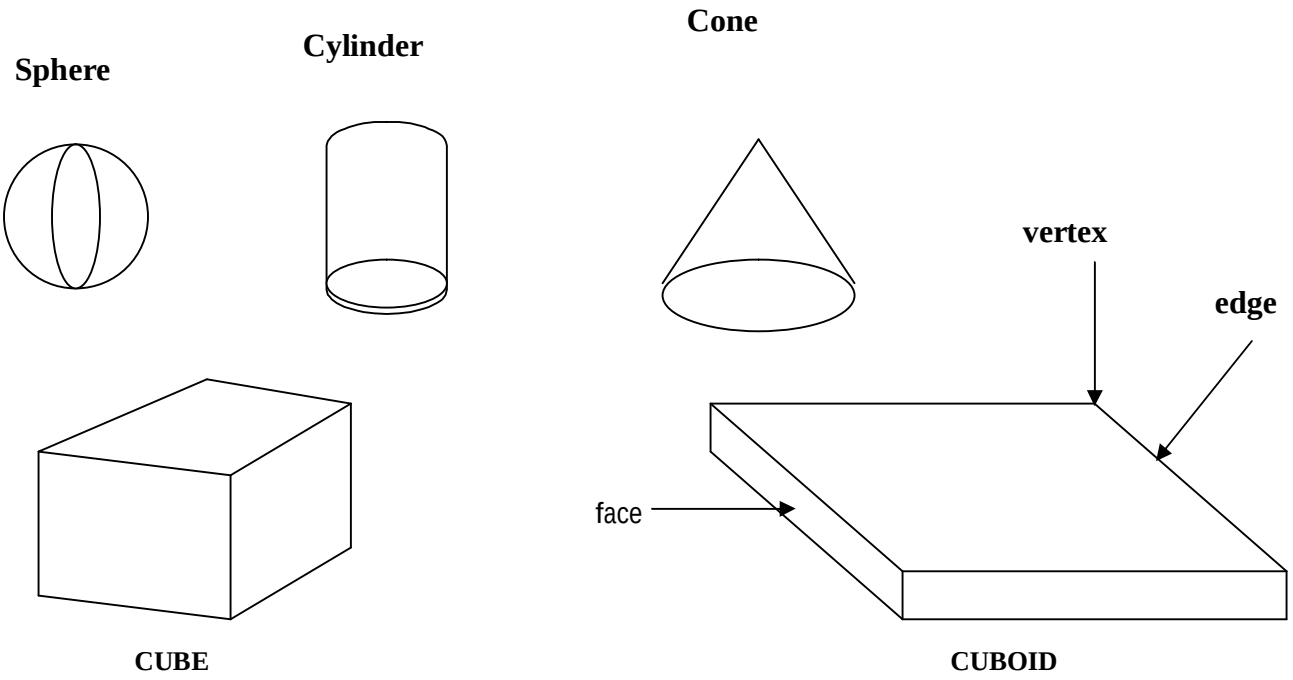
    - § Several different types of materials should be used: e.g., paper, Popsicle sticks, and drinking straws.

    - § Before students make models of three-dimensional shapes, they should engage in activities in which they deconstruct and rebuild given models to note how the parts fit together. Students should also construct shapes by combining given regular shapes.

6. The activities should be accompanied by discussion of the results of the tasks. This discussion should be used to introduce terms so that the students can develop an understanding of the language associated with geometry.

Three-dimensional shapes

The three dimensional shapes under consideration are: spheres, cylinders, cones, cubes and cuboids



All three-dimensional shapes, except the sphere, can be open.

An open shape indicates that a face is missing.

A closed shape means that all faces are included.

Names	Faces	Edges	Vertices
Sphere	1	0	0
Cylinder			
open	2	1	0
closed	3	2	0
Cone			
open	1	0	1
closed	2	1	1
Cube			
open	5	8	8
closed	6	12	8
Cuboid			
open	5	8	8
closed	6	12	8

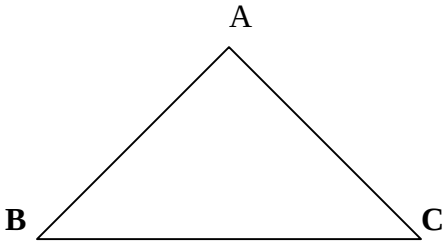
A net of a three-dimensional shape is an ordered arrangement of the faces of the solid on a plane surface:

Plane shapes

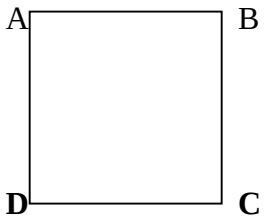
The two dimensional shapes to be studied at this level are triangles and quadrilaterals (squares and rectangles). The circle will also be investigated.

Two-dimensional shapes are bounded by 3 or more line segments called sides.

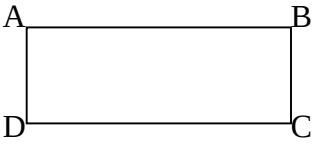
A **triangle** is a closed figure that is made up of 3 sides. Each pair of sides has a common endpoint or vertex. Each pair of sides meet to form an angle.



A **square** has four sides and four angles. All the sides are equal in length. That is opposite and adjacent sides are equal in length.



A **rectangle** has four sides and four angles. Opposite sides are equal in length.



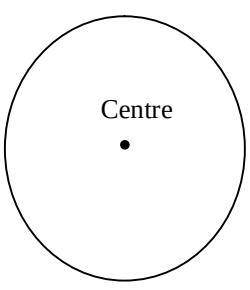
A square is a rectangle but a rectangle is not necessarily a square.

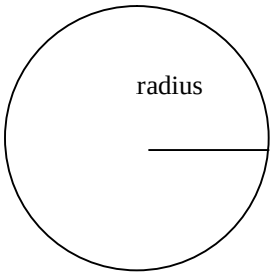
A **circle** is a plane shape with a circumference which is a line segment that is the same distance from the centre at all points.

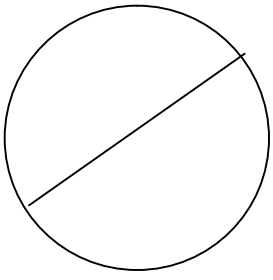
Other parts of a circle are the:

Radius – a line segment that joins the centre of the circle to any part of the circumference.

Diameter – A line segment that passes through the centre of the circle and joins two points on the circumference



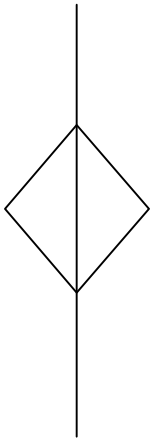




Diameter

A **line of symmetry** divides a shape, such that the images of the object on either side of the line have the same size and shape.
Although shape, size and distance are the same on each side of the line of symmetry, the orientation of the two parts of the object are reversed.

Example of a symmetrical shape.



A **point** is the simplest of all geometric figures. Points are represented by dots and labeled with upper-case letters. E.g., point A may be represented as follows.

. A

A **line segment** is the shortest distance between two points. A line segment is made up of a set of points, and has two endpoints.
A **line** is the extension of a line segment in both directions.
Lines and line segments are represented as follows. Any two points on a line or line segment may be used to label or name it.

Line segment AB **Line CD**

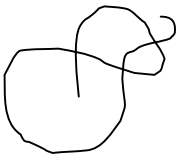
Lines that meet or cross each other are **intersecting lines**.
Horizontal lines run in a left-right direction.
Vertical lines run in a north-south direction.
When horizontal and vertical lines intersect, they form right angles.

A **curve** is a path that can be traced without lifting your pencil.
It is a **simple curve** if the tracing does not go through any point more than once.

An **open curve** has endpoints. A **closed curve** has no endpoints.

A closed simple curve

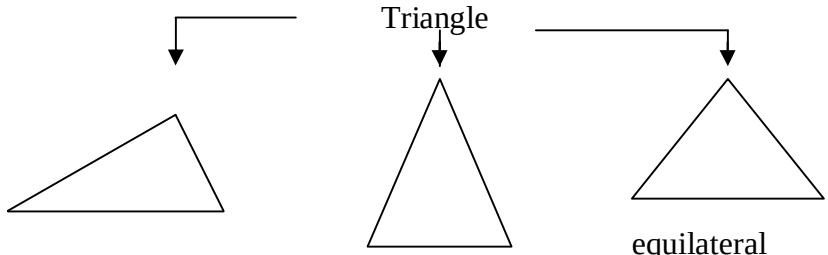
An open simple curve



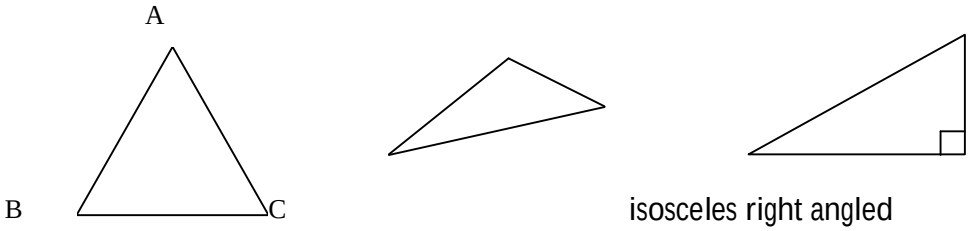
An open curve that is not simple

An **angle** is formed when two lines or line segments meet.
The measure or size of an angle is the amount of turn between the two lines or line segments.
A **right angle** is formed when a vertical and a horizontal line meet.

Classification of triangles



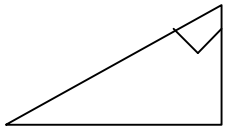
acute angled obtuse angled right angled



isosceles right angled

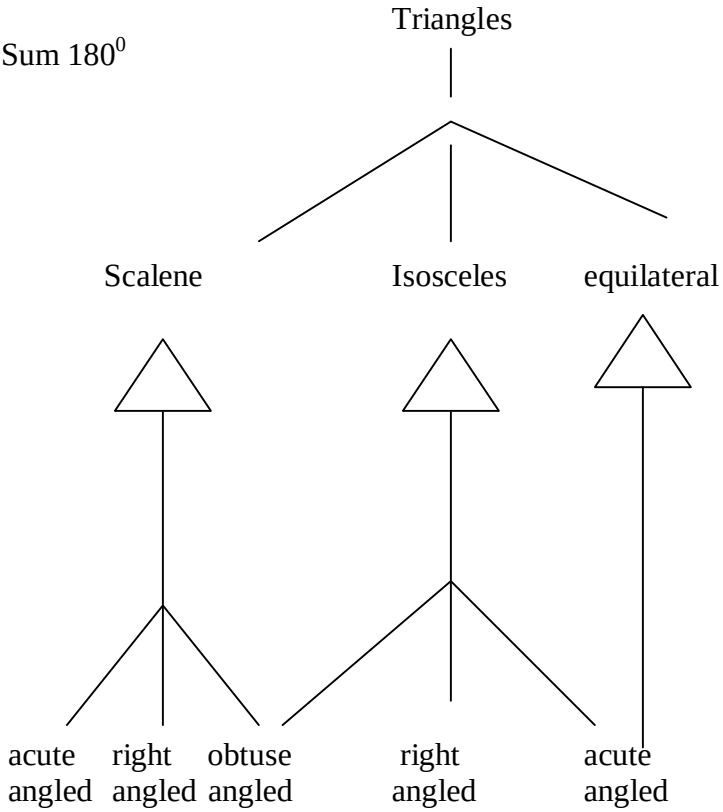
Â, B, and C
All less that 90^0

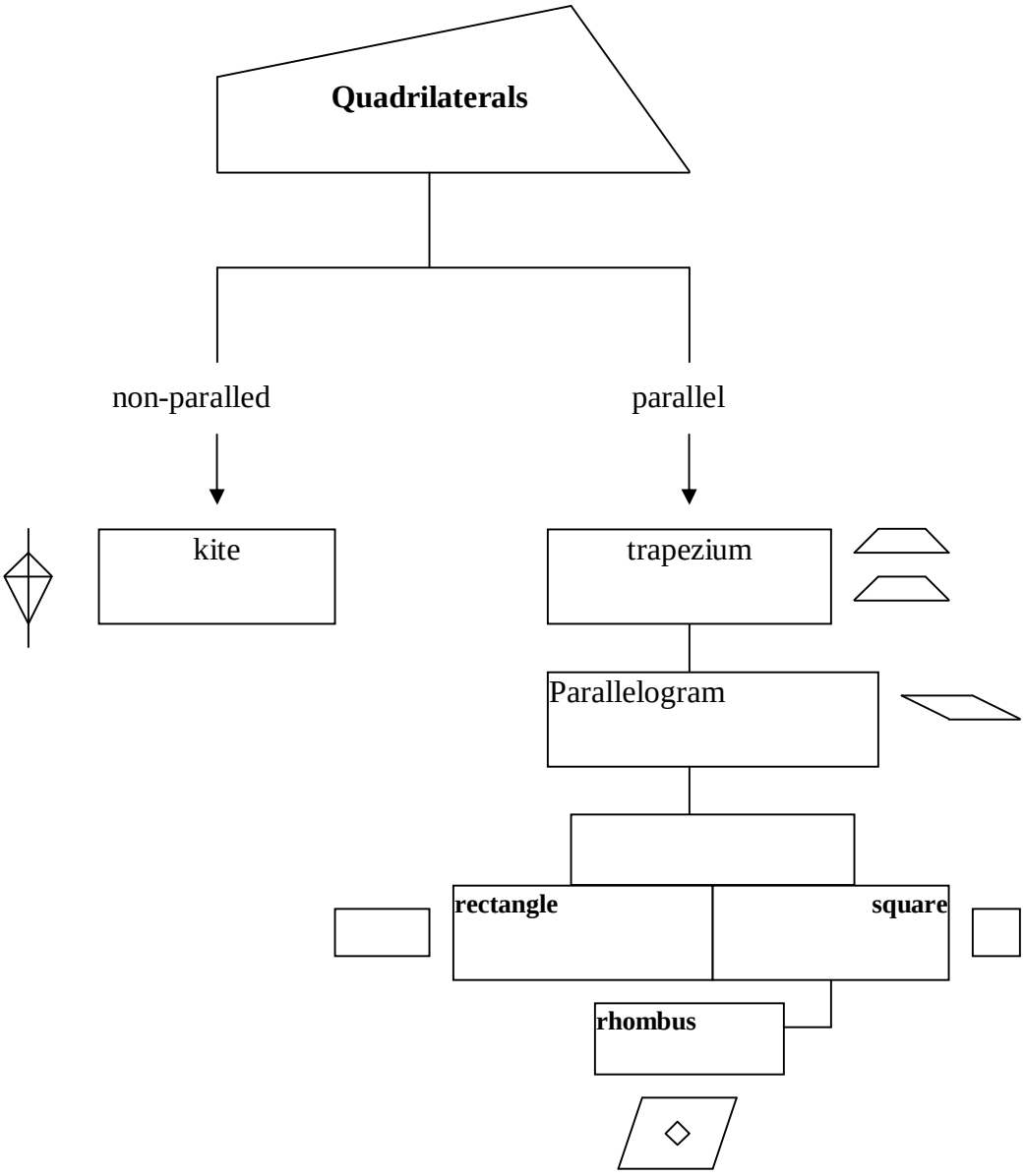
one angle
$90^0 < x^0 < 180^0$



One angle 90^0

3 sides }
3 angles } Sum 180^0




Quadrilaterals
non-paralled
parallel
kite
trapezium
Parallelogram
rectangle
square
rhombus

Specific Activities – Grade 6

Outcome 6

Students should be able to:

Draw sketches of 3-D, shapes from different perspectives.

Materials: Models of three-dimensional shapes

Activity: Give each student an example of a cube.

Ask the students to place the cube on their desks, to look down on the cube and draw it from the top view. Check the students' drawings.

Guide the students to state where they would need to stand to obtain a side view of the cube. Check that the students' responses are correct and let them draw a side view of the cube.

Continue by having the students draw the front or back view.

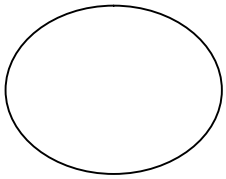
Repeat the activity, using other three-dimensional shapes – cuboid, cone, cylinder cone and sphere.

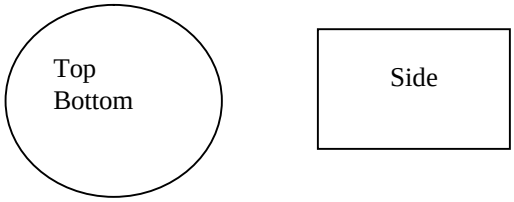
Encourage the students to compare and contrast the perspective drawings for the cube and cuboid, and the cone and cylinder

Note that these are the possible results/drawings

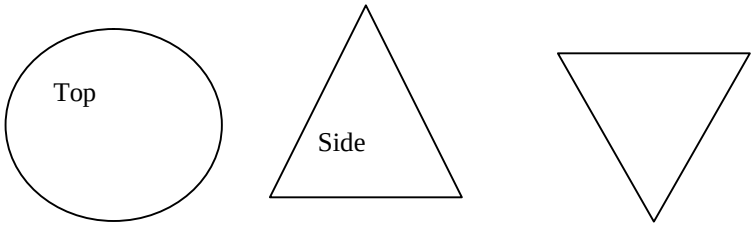
Cubes  all views

Cuboids 

Sphere  all views

Cylinder 

Cone



Outcome 15

Students should be able to:

Draw two-dimensional shapes according to directions that are based on geometric concepts and the properties of the shapes, e.g., symmetry, type of figure (open or closed), number of sides, types of sides (parallel or perpendicular), etc.

Materials: Paper, pencils; work sheet containing descriptions of two-dimensional shapes.

Activities: As a class activity, give the students a description of a shape and ask them to draw the shape.
For example, the description may be:

'*Draw a plane shape that has four lines of symmetry.'*
You may also provide the students with a description of a shape and ask them if the shape exists.
For example: '*Can you draw an equilateral triangle that is right angled?'*

Ask the students to draw the shapes described on the worksheet.

Ask the students to make up related descriptions/tasks and have them rate them as 'challenging' or 'not challenging'. Have them explain why they gave the description/task the rating that they did.